Revisiting Beimel-Weinreb Weighted Threshold Secret Sharing Schemes

Computer Security Foundations Symposium 2026, Lisboa

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Universitat Rovira i Virgili

28 July 2026

Introduction: Secret Sharing Schemes

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Cryptographic primitive that allows a dealer to **share** a secret among a set of parties, so that only **authorized** subsets of them can **recover** it.

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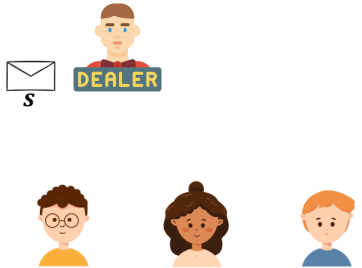

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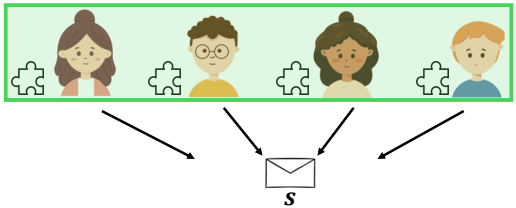

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- Correctness
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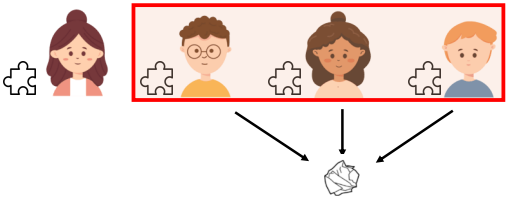

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- Access structures are expressed as monotone Boolean functions $f : \{0,1\}^n \to \{0,1\}$.

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$$f(\text{  }, \text{  }, \text{  }) = 0$$

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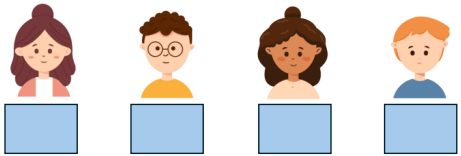

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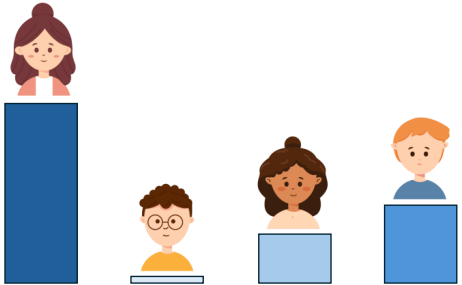

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A subset X is authorized if and only if $|X| \geq t$. Γ is given by $\boldsymbol{f(x) = \mathrm{sign}(x_1 + \ldots + x_n - t)}$.

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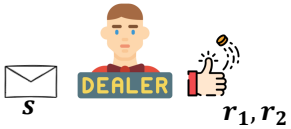

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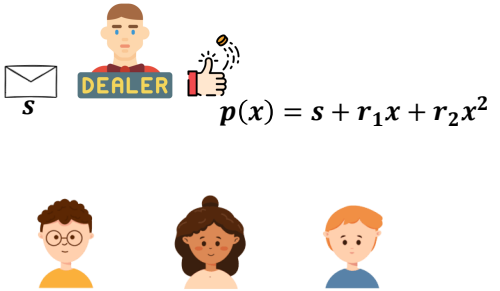

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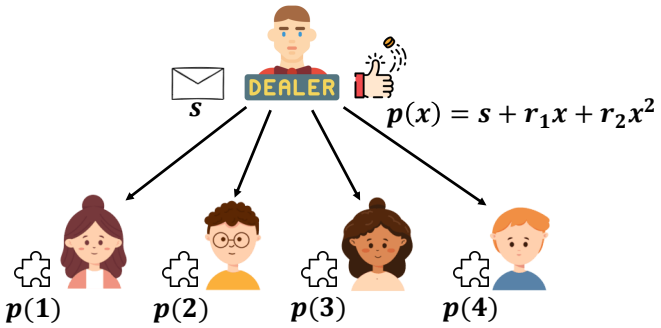

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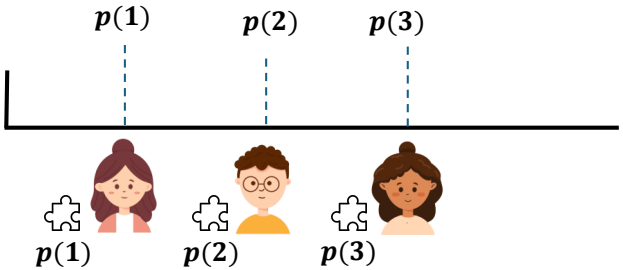

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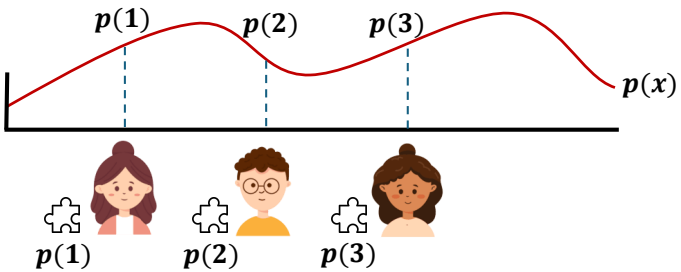

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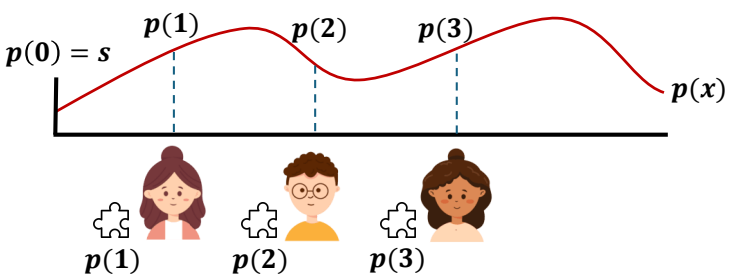

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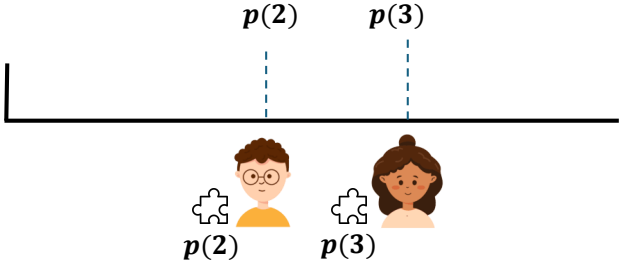

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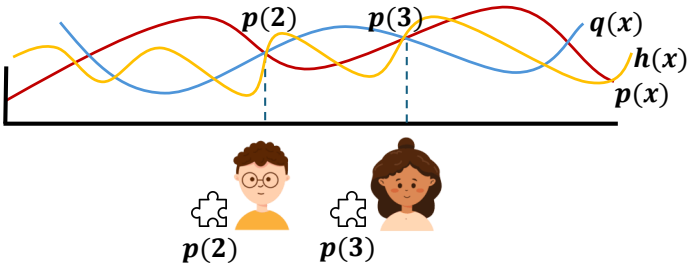

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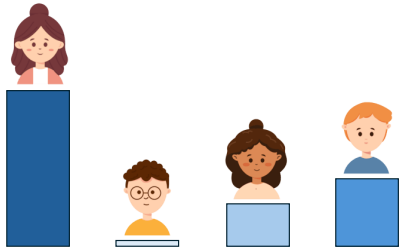

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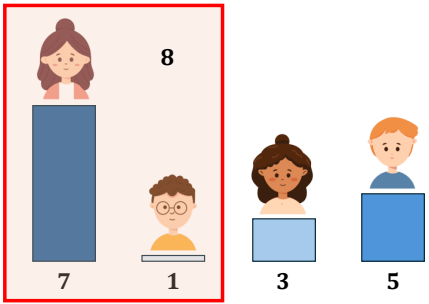

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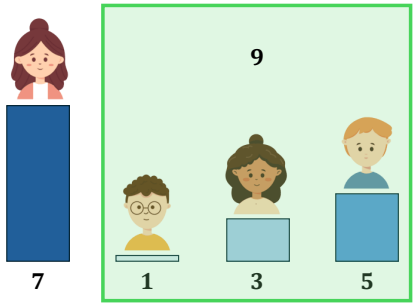

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There exists a WTAS requiring weights of exponential size.

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- Shamir scheme leads to exponential share size!

Introduction: Weighted Threshold State-of-the-Art

Question: Is there any alternative construction that reduces the share size?

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For nearly 20 years, these have been the most efficient constructions.

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Problem

The circuit requires sorting gates that are not feasible in practice.

Introduction: Goal

Question: Can we **improve** the state-of-the-art **weighted threshold** secret sharing schemes?

Introduction: Our Results

This Work

- A new **monotone circuit** for **WTF**
- An **efficient** computational scheme for **WTAS**
- An **information-theoretic** scheme for **ramp WTAS**

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Main Result: **Computational scheme for WTAS**

- Share size $\boldsymbol{\lambda}$
- Public information size $\boldsymbol{4\lambda n^2 \log \sigma}$
- Usage of only $\boldsymbol{4\lambda n \log \sigma}$ bits of the public information during reconstruction

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Comparison

$300\times - 6700\times$ smaller shares than the previous schemes for deployed blockchain networks.

Revisiting Beimel & Weinreb Schemes: Their Construction

Theorem [Beimel, Weinreb'06]

Given a security parameter $\lambda \in \mathbb{N}$, WTASs admit **computational** schemes with

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$$f(x)$$

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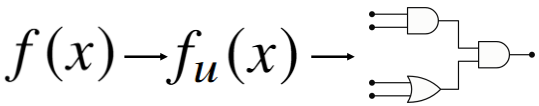

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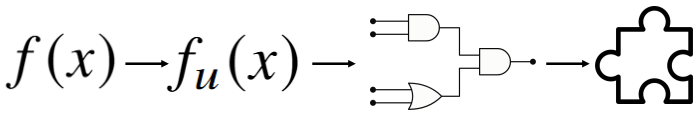

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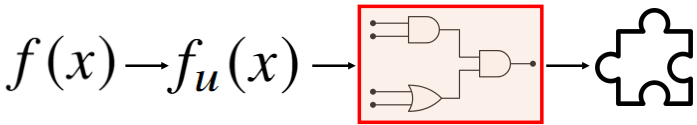

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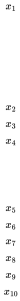

Revisiting Beimel & Weinreb Schemes: Our Circuit

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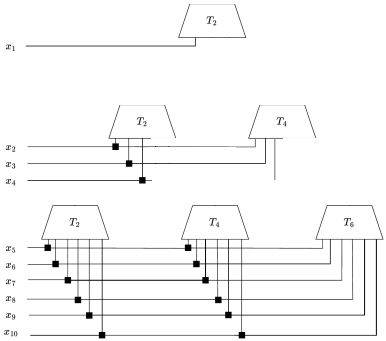

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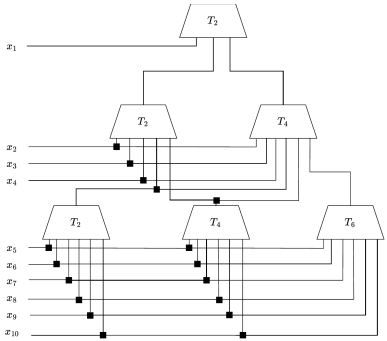

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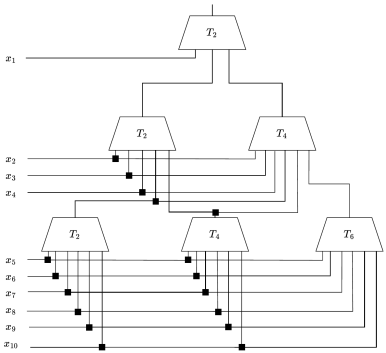

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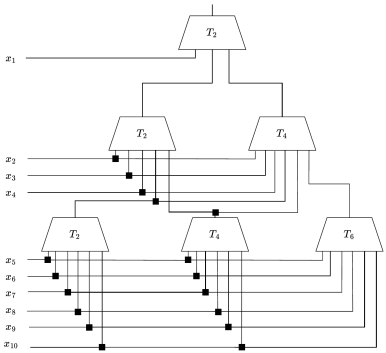


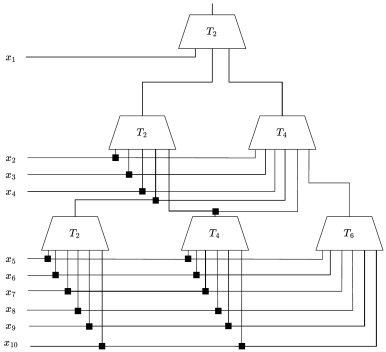
Theorem

This circuit has $2n^2 \log \sigma$ wires.

Revisiting Beimel & Weinreb Schemes: Optimization

$$f(x) = \mathrm{sign}(4x_1 + 2x_2 + 2x_3 + 2x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} - 8)$$

Idea: To **reduce** the number of **used wires** by adding **auxiliary OR** gates.



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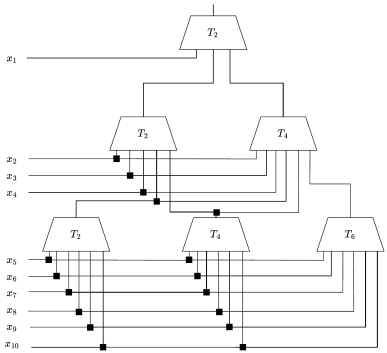

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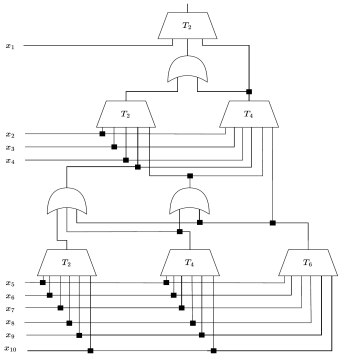

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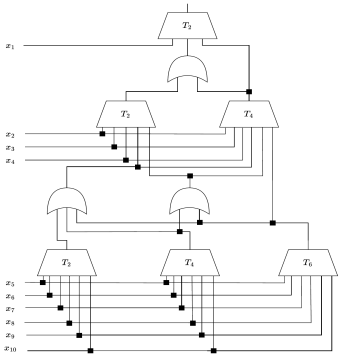

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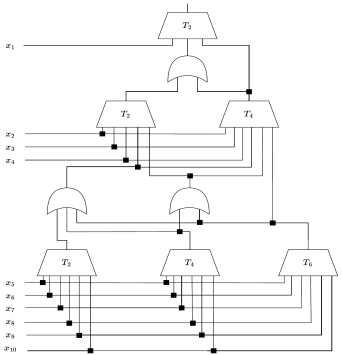

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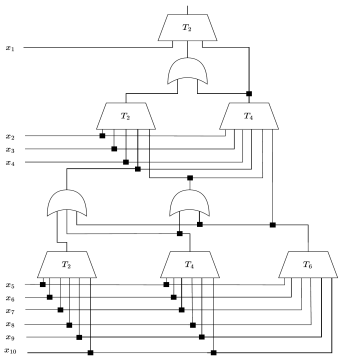

Theorem

Computing the output from any given input requires only $4n \log \sigma$ wires.

Revisiting Beimel & Weinreb Schemes: Our Results

Theorem

Given a security parameter $\lambda \in \mathbb{N}$, WTASs admit **computational** schemes with

- share size $\boldsymbol{\lambda}$
- public information size $\boldsymbol{4\lambda n^2 \log \sigma}$
- usage of only $\boldsymbol{4\lambda n \log \sigma}$ bits of the public information during reconstruction

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Proof: Apply **Yao's compiler** to our circuit.

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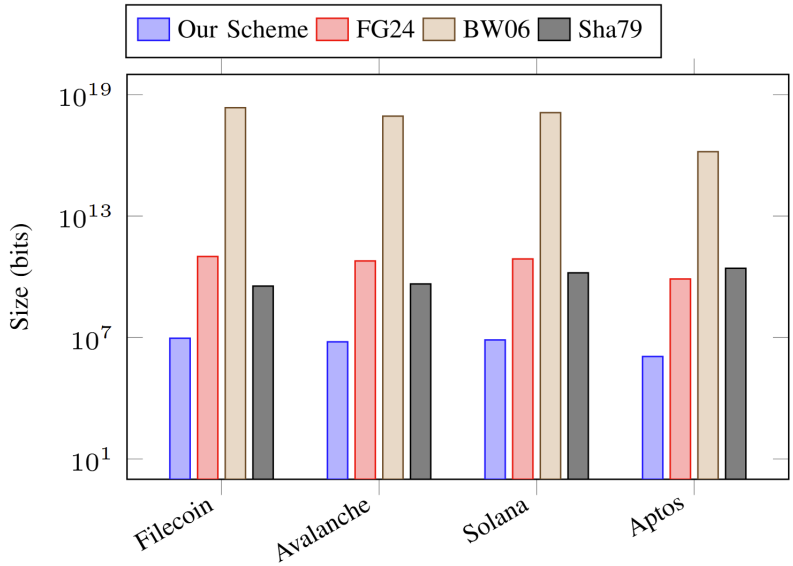
Information-Theoretic Scheme

Applying balancing and dynamic programming to our circuit yields an equivalent information-theoretic scheme to that of Beimel and Weinreb.

Revisiting Beimel & Weinreb Schemes: Comparison



Closing Remarks & Open Questions

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- The **share size** of existing schemes for WTASs have a **strong dependency** on the **size of the weights**.
- The most efficient **information-theoretic** scheme has **superpolynomial** share size.
- Moving towards the **computational** setting allows more efficient schemes.
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Open Questions

- Is there any scheme with **polynomial** share size for WTASs?
- Can we improve the $\boldsymbol{\Omega(\sqrt{n})}$ **lower bound** for these access structures?
- Can we make our scheme **linear**?
- Are weighted threshold functions in $\mathbf{mNC^1}$?

Moltes Gràcies!

Check the Full Version of our Paper!